A simulation study to examine the information content in phylogenomic datasets under the multispecies coalescent model

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Supplementary material Supplementary material


Figure S1: The posterior $95 \%$ CIs and CI coverage for parameters under the MSC model for species tree U of figure 1. Model parameters for unbalanced tree: $\tau_{R}=5 \theta, \tau_{S}=4 \theta, \tau_{T}=3 \theta, \tau_{U}=2.5 \theta$. See legend for figure 2 .


Figure S2: Posterior 95\% HPD CIs and CI coverage for the 21 parameters in MSci model B of figure 8.


Figure S3: Posterior 95\% HPD CIs and CI coverage for the 21 parameters in MSci model U of figure 8.
Table S1: Average CI width for the 13 parameters in the MSC models of figure $1\left(\times 10^{-3}\right)$

|  |  | $\theta_{A}$ | $\theta_{B}$ | $\theta_{C}$ | $\theta_{D}$ | $\theta_{E}$ | $\theta_{R}$ | $\theta_{S}$ | $\theta_{T}$ | $\theta_{U}$ | $\tau_{R}$ | $\tau_{S}$ | $\tau_{T}$ | $\tau_{U}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tree B |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $L=40, N=250$ | 2.19 | 2.28 | 2.25 | 2.23 | 2.26 | 3.23 | 4.33 | 5.21 | 7.86 | 2.69 | 2.87 | 3.05 | 3.88 |
| $\begin{aligned} & S=2 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=1000$ | 1.82 | 1.77 | 1.72 | 1.73 | 1.75 | 2.43 | 3.27 | 3.46 | 6.60 | 1.45 | 1.65 | 1.71 | 2.38 |
|  | $L=160, N=250$ | 1.19 | 1.21 | 1.20 | 1.19 | 1.20 | 2.47 | 3.35 | 3.86 | 7.75 | 1.57 | 1.74 | 1.87 | 2.68 |
|  | $L=160, N=1000$ | 0.90 | 0.91 | 0.89 | 0.89 | 0.90 | 1.39 | 1.99 | 1.96 | 4.75 | 0.76 | 0.91 | 0.91 | 1.53 |
| $\begin{aligned} & S=8 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=250$ | 1.31 | 1.31 | 1.33 | 1.34 | 1.33 | 3.32 | 4.50 | 4.78 | 7.70 | 2.66 | 2.82 | 2.85 | 3.67 |
|  | $L=40, N=1000$ | 0.88 | 0.88 | 0.89 | 0.88 | 0.88 | 2.47 | 3.19 | 3.15 | 6.97 | 1.43 | 1.61 | 1.60 | 2.38 |
|  | $L=160, N=250$ | 0.68 | 0.68 | 0.68 | 0.69 | 0.69 | 2.42 | 3.41 | 3.42 | 7.22 | 1.53 | 1.70 | 1.71 | 2.51 |
|  | $L=160, N=1000$ | 0.44 | 0.44 | 0.45 | 0.45 | 0.45 | 1.38 | 1.94 | 1.90 | 5.04 | 0.74 | 0.89 | 0.87 | 1.55 |
| $\begin{aligned} & S=2 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 7.04 | 6.97 | 6.96 | 7.02 | 7.05 | 10.06 | 14.46 | 14.51 | 28.18 | 6.04 | 7.02 | 7.11 | 10.05 |
|  | $L=40, N=1000$ | 6.44 | 6.27 | 6.32 | 6.25 | 6.33 | 7.28 | 9.86 | 9.52 | 20.01 | 3.28 | 3.93 | 3.89 | 6.07 |
|  | $L=160, N=250$ | 3.58 | 3.55 | 3.59 | 3.62 | 3.61 | 5.83 | 8.39 | 8.61 | 20.20 | 3.17 | 3.86 | 3.93 | 6.45 |
|  | $L=160, N=1000$ | 3.22 | 3.19 | 3.23 | 3.19 | 3.19 | 3.90 | 5.15 | 5.26 | 12.29 | 1.67 | 1.99 | 2.01 | 3.61 |
| $\begin{aligned} & S=8 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 3.50 | 3.52 | 3.51 | 3.56 | 3.50 | 10.06 | 13.34 | 13.76 | 26.41 | 5.93 | 6.70 | 6.71 | 9.65 |
|  | $L=40, N=1000$ | 2.75 | 2.74 | 2.72 | 2.73 | 2.74 | 7.27 | 9.45 | 9.65 | 18.46 | 3.19 | 3.82 | 3.77 | 5.72 |
|  | $L=160, N=250$ | 1.77 | 1.77 | 1.78 | 1.80 | 1.79 | 5.74 | 8.28 | 8.20 | 19.25 | 3.09 | 3.75 | 3.69 | 6.20 |
|  | $L=160, N=1000$ | 1.38 | 1.37 | 1.38 | 1.37 | 1.38 | 3.88 | 5.03 | 5.06 | 11.80 | 1.64 | 1.94 | 1.92 | 3.46 |
| Tree U |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | $L=40, N=250$ | 2.30 | 2.24 | 2.22 | 2.29 | 2.20 | 4.09 | 4.45 | 3.38 | 6.30 | 3.34 | 2.75 | 2.23 | 2.67 |
| $\begin{aligned} & S=2 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=1000$ | 1.72 | 1.75 | 1.77 | 1.67 | 1.75 | 3.06 | 3.30 | 2.53 | 4.99 | 1.94 | 1.62 | 1.23 | 1.63 |
|  | $L=160, N=250$ | 1.18 | 1.18 | 1.18 | 1.19 | 1.19 | 3.19 | 3.28 | 2.38 | 5.26 | 2.09 | 1.67 | 1.25 | 1.72 |
|  | $L=160, N=1000$ | 0.89 | 0.89 | 0.89 | 0.90 | 0.90 | 1.99 | 1.93 | 1.47 | 3.03 | 1.12 | 0.87 | 0.66 | 0.93 |
| $\begin{aligned} & S=8 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=250$ | 1.33 | 1.32 | 1.31 | 1.34 | 1.34 | 4.02 | 4.53 | 3.41 | 5.76 | 3.29 | 2.68 | 2.11 | 2.41 |
|  | $L=40, N=1000$ | 0.87 | 0.86 | 0.88 | 0.89 | 0.89 | 3.05 | 3.24 | 2.50 | 4.46 | 1.93 | 1.56 | 1.18 | 1.48 |
|  | $L=160, N=250$ | 0.68 | 0.68 | 0.68 | 0.69 | 0.70 | 3.13 | 3.28 | 2.29 | 4.65 | 2.04 | 1.63 | 1.18 | 1.52 |
|  | $L=160, N=1000$ | 0.44 | 0.44 | 0.44 | 0.45 | 0.45 | 1.96 | 1.93 | 1.46 | 2.80 | 1.09 | 0.85 | 0.64 | 0.86 |
| $\begin{aligned} & S=2 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 7.04 | 7.08 | 6.98 | 6.76 | 6.72 | 12.32 | 13.50 | 10.51 | 20.47 | 8.03 | 6.71 | 5.14 | 6.65 |
|  | $L=40, N=1000$ | 6.34 | 6.49 | 6.27 | 6.27 | 6.42 | 8.94 | 9.46 | 8.01 | 13.84 | 4.53 | 3.78 | 2.95 | 3.84 |
|  | $L=160, N=250$ | 3.62 | 3.59 | 3.59 | 3.56 | 3.60 | 8.45 | 8.29 | 6.20 | 12.91 | 4.71 | 3.76 | 2.78 | 3.93 |
|  | $L=160, N=1000$ | 3.25 | 3.21 | 3.18 | 3.18 | 3.23 | 4.78 | 5.14 | 4.21 | 7.67 | 2.28 | 1.94 | 1.51 | 2.06 |
| $\begin{aligned} & S=8 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 3.52 | 3.52 | 3.57 | 3.56 | 3.54 | 12.73 | 14.34 | 10.66 | 18.47 | 7.97 | 6.62 | 4.96 | 6.13 |
|  | $L=40, N=1000$ | 2.71 | 2.75 | 2.73 | 2.75 | 2.72 | 9.13 | 9.60 | 8.06 | 13.60 | 4.45 | 3.71 | 2.86 | 3.68 |
|  | $L=160, N=250$ | 1.78 | 1.78 | 1.79 | 1.79 | 1.80 | 8.15 | 8.35 | 6.15 | 11.98 | 4.56 | 3.63 | 2.66 | 3.60 |
|  | $L=160, N=1000$ | 1.36 | 1.38 | 1.38 | 1.38 | 1.38 | 4.77 | 5.00 | 4.20 | 7.13 | 2.24 | 1.87 | 1.46 | 1.91 |

Table S2: Root mean square error (RMSE) for the 13 parameters in the MSC models of figure $1\left(\times 10^{-3}\right)$

Table S3: Average 95\% HPD CI width for the 21 parameters in the MSci models of figure $8\left(\times 10^{-3}\right.$ for $\theta \mathrm{s}$ and $\left.\tau \mathrm{s}\right)$

|  |  | $\theta_{A}$ | $\theta_{B}$ | $\theta_{C}$ | $\theta_{D}$ | $\theta_{E}$ | $\theta_{R}$ | $\theta_{S}$ | $\theta_{T}$ | $\theta_{U}$ | $\theta_{X}$ | $\theta_{Y}$ | $\theta_{W}$ | $\theta_{Z}$ | $\tau_{R}$ | $\tau_{S}$ | $\tau_{T}$ | $\tau_{U}$ | $\tau_{X}$ | $\tau_{W}$ | $\varphi_{Y}$ | $\varphi_{Z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\theta=0.0025$ | $L=40, N=1000$ | 1.74 | 2.06 | 2.00 | 2.09 | 2.04 | 2.65 | 3.57 | 3.43 | 7.37 | 3.24 | 4.60 | 3.59 | 4.69 | 1.57 | 1.92 | 1.73 | 2.76 | 1.51 | 1.88 | 0.265 | 0.236 |
|  | $L=160, N=250$ | 1.19 | 1.44 | 1.36 | 1.46 | 1.43 | 2.75 | 3.62 | 3.76 | 8.37 | 3.15 | 4.91 | 3.31 | 4.60 | 1.73 | 2.10 | 1.89 | 3.32 | 1.39 | 1.86 | 0.178 | 0.180 |
|  | $L=160, N=1000$ | 0.90 | 1.00 | 1.00 | 1.01 | 1.02 | 1.53 | 2.26 | 2.16 | 5.86 | 1.99 | 3.63 | 2.28 | 3.33 | 0.83 | 1.09 | 0.97 | 1.84 | 0.79 | 0.96 | 0.138 | 0.121 |
| $\begin{aligned} & S=8 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=250$ | 1.33 | 1.51 | 1.43 | 1.49 | 1.50 | 3.40 | 4.58 | 4.75 | 8.34 | 3.52 | 4.93 | 3.89 | 4.29 | 2.78 | 3.22 | 2.90 | 4.46 | 1.93 | 2.59 | 0.303 | 0.302 |
|  | $L=40, N=1000$ | 0.88 | 0.94 | 0.94 | 0.94 | 0.95 | 2.54 | 3.60 | 3.35 | 7.09 | 2.65 | 4.18 | 2.72 | 3.59 | 1.51 | 1.84 | 1.66 | 2.63 | 1.09 | 1.40 | 0.253 | 0.218 |
|  | $L=160, N=250$ | 0.68 | 0.76 | 0.74 | 0.77 | 0.76 | 2.66 | 3.66 | 3.52 | 7.54 | 2.27 | 4.07 | 2.48 | 3.35 | 1.68 | 2.01 | 1.78 | 2.99 | 0.95 | 1.20 | 0.157 | 0.151 |
|  | $L=160, N=1000$ | 0.44 | 0.47 | 0.47 | 0.48 | 0.48 | 1.47 | 2.24 | 2.04 | 4.74 | 1.46 | 2.75 | 1.58 | 2.30 | 0.80 | 1.04 | 0.91 | 1.62 | 0.56 | 0.69 | 0.130 | 0.112 |
| $\begin{aligned} & S=2 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 6.85 | 8.14 | 8.22 | 8.18 | 8.32 | 10.88 | 14.96 | 15.07 | 31.33 | 13.23 | 19.07 | 15.30 | 17.15 | 6.51 | 8.06 | 7.37 | 11.77 | 6.11 | 7.76 | 0.272 | 0.240 |
|  | $L=40, N=1000$ | 6.31 | 7.10 | 7.23 | 7.15 | 7.01 | 7.93 | 10.91 | 10.30 | 20.44 | 10.55 | 18.32 | 11.88 | 17.56 | 3.55 | 4.66 | 4.07 | 6.57 | 3.86 | 4.78 | 0.262 | 0.227 |
|  | $L=160, N=250$ | 3.59 | 4.03 | 4.04 | 4.10 | 4.09 | 6.32 | 9.84 | 9.18 | 21.79 | 8.31 | 15.16 | 9.38 | 13.47 | 3.45 | 4.70 | 4.10 | 7.23 | 3.21 | 3.85 | 0.140 | 0.124 |
|  | $L=160, N=1000$ | 3.20 | 3.57 | 3.57 | 3.58 | 3.54 | 4.18 | 5.78 | 5.36 | 13.49 | 5.98 | 12.83 | 6.88 | 11.13 | 1.81 | 2.34 | 2.06 | 4.01 | 1.98 | 2.40 | 0.134 | 0.116 |
| $\begin{aligned} & S=8 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 3.51 | 3.70 | 3.69 | 3.80 | 3.75 | 10.56 | 14.16 | 14.61 | 27.90 | 10.70 | 16.77 | 10.60 | 15.18 | 6.26 | 7.57 | 6.95 | 10.52 | 4.57 | 5.60 | 0.254 | 0.217 |
|  | $L=40, N=1000$ | 2.74 | 2.85 | 2.82 | 2.83 | 2.81 | 7.65 | 10.45 | 9.71 | 17.91 | 8.64 | 14.23 | 9.56 | 14.03 | 3.43 | 4.38 | 3.82 | 5.98 | 3.01 | 3.62 | 0.243 | 0.213 |
|  | $L=160, N=250$ | 1.78 | 1.88 | 1.86 | 1.90 | 1.89 | 6.23 | 9.47 | 8.75 | 20.55 | 5.84 | 11.39 | 6.37 | 9.99 | 3.35 | 4.37 | 3.86 | 6.84 | 2.26 | 2.75 | 0.131 | 0.113 |
|  | $L=160, N=1000$ | 1.38 | 1.43 | 1.42 | 1.43 | 1.42 | 4.08 | 5.56 | 5.17 | 12.41 | 4.52 | 8.68 | 5.02 | 7.91 | 1.75 | 2.23 | 1.94 | 3.74 | 1.47 | 1.79 | 0.125 | 0.109 |
| Tree U |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $S=2$ | $L=40, N=250$ | 2.25 | 2.94 | 2.64 | 3.08 | 2.84 | 4.48 | 4.65 | 4.08 | 6.17 | 4.80 | 5.45 | 4.43 | 4.92 | 3.97 | 2.85 | 3.11 | 2.63 | 2.97 | 3.74 | 0.493 | 0.316 |
| $\theta=0.0025$ | $L=40, N=1000$ | 1.74 | 2.08 | 1.94 | 2.06 | 2.04 | 3.11 | 3.73 | 2.78 | 4.54 | 3.61 | 4.94 | 3.42 | 4.78 | 2.14 | 1.74 | 1.53 | 1.60 | 1.50 | 1.81 | 0.305 | 0.232 |
|  | $L=160, N=250$ | 1.19 | 1.47 | 1.38 | 1.52 | 1.38 | 3.47 | 3.67 | 2.82 | 5.22 | 3.52 | 4.91 | 3.16 | 4.61 | 2.40 | 1.80 | 1.68 | 1.75 | 1.50 | 1.69 | 0.254 | 0.154 |
|  | $L=160, N=1000$ | 0.88 | 1.02 | 1.01 | 1.00 | 1.02 | 2.22 | 2.14 | 1.72 | 3.24 | 2.29 | 3.88 | 2.19 | 3.53 | 1.28 | 0.96 | 0.83 | 0.98 | 0.79 | 0.95 | 0.156 | 0.120 |
| $\begin{aligned} & S=8 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=250$ | 1.34 | 1.48 | 1.46 | 1.52 | 1.53 | 4.30 | 4.53 | 4.05 | 6.09 | 3.98 | 5.48 | 3.63 | 4.61 | 3.71 | 2.76 | 2.87 | 2.49 | 2.12 | 2.30 | 0.425 | 0.261 |
|  | $L=40, N=1000$ | 0.86 | 0.93 | 0.92 | 0.94 | 0.93 | 3.31 | 3.48 | 2.69 | 4.62 | 2.99 | 4.46 | 2.84 | 3.93 | 2.11 | 1.66 | 1.41 | 1.50 | 1.15 | 1.39 | 0.278 | 0.215 |
|  | $L=160, N=250$ | 0.68 | 0.75 | 0.75 | 0.76 | 0.76 | 3.30 | 3.53 | 2.79 | 4.87 | 2.75 | 4.71 | 2.47 | 3.29 | 2.25 | 1.73 | 1.57 | 1.60 | 1.06 | 1.13 | 0.219 | 0.136 |
|  | $L=160, N=1000$ | 0.44 | 0.47 | 0.46 | 0.47 | 0.47 | 2.12 | 2.11 | 1.62 | 3.07 | 1.58 | 2.86 | 1.58 | 2.39 | 1.20 | 0.92 | 0.76 | 0.90 | 0.57 | 0.68 | 0.140 | 0.112 |
| $\begin{aligned} & S=2 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 6.96 | 8.20 | 7.79 | 8.29 | 8.18 | 13.29 | 14.80 | 11.97 | 22.22 | 15.05 | 20.29 | 13.65 | 19.10 | 8.81 | 7.06 | 6.68 | 6.97 | 6.37 | 7.68 | 0.316 | 0.229 |
|  | $L=40, N=1000$ | 6.25 | 7.15 | 7.22 | 7.18 | 7.17 | 9.77 | 9.82 | 9.27 | 14.46 | 12.17 | 18.63 | 11.48 | 17.31 | 4.99 | 3.97 | 3.74 | 3.97 | 3.81 | 4.90 | 0.275 | 0.219 |
|  | $L=160, N=250$ | 3.59 | 4.13 | 4.00 | 4.09 | 4.08 | 9.08 | 9.22 | 7.23 | 14.06 | 9.19 | 15.25 | 9.00 | 13.68 | 5.22 | 4.05 | 3.51 | 4.13 | 3.21 | 3.86 | 0.159 | 0.120 |
|  | $L=160, N=1000$ | 3.21 | 3.54 | 3.54 | 3.56 | 3.57 | 5.23 | 5.25 | 4.91 | 8.13 | 6.80 | 13.43 | 6.44 | 10.81 | 2.53 | 2.03 | 1.89 | 2.15 | 1.96 | 2.42 | 0.140 | 0.115 |
| $\begin{aligned} & S=8 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 3.46 | 3.78 | 3.73 | 3.77 | 3.76 | 12.28 | 13.95 | 11.76 | 18.71 | 11.51 | 18.06 | 11.04 | 15.17 | 8.28 | 6.83 | 5.97 | 6.16 | 4.66 | 5.40 | 0.281 | 0.218 |
|  | $L=40, N=1000$ | 2.74 | 2.85 | 2.82 | 2.88 | 2.88 | 9.66 | 10.08 | 8.80 | 13.11 | 9.69 | 16.76 | 9.21 | 13.20 | 4.89 | 3.88 | 3.39 | 3.63 | 3.01 | 3.56 | 0.253 | 0.210 |
|  | $L=160, N=250$ | 1.77 | 1.90 | 1.86 | 1.89 | 1.88 | 8.73 | 8.65 | 6.83 | 12.69 | 6.44 | 12.60 | 6.45 | 9.58 | 4.96 | 3.88 | 3.19 | 3.75 | 2.30 | 2.70 | 0.143 | 0.113 |
|  | $L=160, N=1000$ | 1.37 | 1.42 | 1.42 | 1.43 | 1.43 | 5.09 | 5.26 | 4.59 | 7.57 | 5.06 | 9.76 | 4.92 | 7.85 | 2.46 | 1.98 | 1.72 | 1.98 | 1.49 | 1.76 | 0.129 | 0.110 |

Table S4: Root mean square error (RMSE) for the 21 parameters in the MSci models of figure $8\left(\times 10^{-3}\right.$ for $\theta \mathrm{s}$ and $\left.\tau \mathrm{s}\right)$

|  |  | $\theta_{A}$ | $\theta_{B}$ | $\theta_{C}$ | $\theta_{D}$ | $\theta_{E}$ | $\theta_{R}$ | $\theta_{S}$ | $\theta_{T}$ | $\theta_{U}$ | $\theta_{X}$ | $\theta_{Y}$ | $\theta_{W}$ | $\theta_{Z}$ | $\tau_{R}$ | $\tau_{S}$ | $\tau_{T}$ | $\tau_{U}$ | $\tau_{X}$ | $\tau_{W}$ | $\varphi_{Y}$ | $\varphi_{Z}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Tree B |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\theta=0.0025$ | $L=40, N=250$ $L=40, N=1000$ | 0.58 | 0.70 | 0.74 0.51 | 0.65 | 0.66 0.52 | 0.73 0.57 | 0.76 0.69 | 0.75 0.76 | 1.53 | 0.66 | 0.65 0.80 | 0.73 | 0.58 0.79 | 0.35 | 0.74 0.44 | 0.41 | 0.99 | 0.75 0.42 | 1.12 | 0.074 | 0.050 0.050 |
|  | $L=160, N=250$ | 0.31 | 0.39 | 0.35 | 0.32 | 0.39 | 0.65 | 0.74 | 0.79 | 1.18 | 0.72 | 0.84 | 0.66 | 0.81 | 0.40 | 0.48 | 0.39 | 0.52 | 0.38 | 0.52 | 0.046 | 0.043 |
|  | $L=160, N=1000$ | 0.23 | 0.29 | 0.24 | 0.25 | 0.27 | 0.35 | 0.51 | 0.57 | 1.13 | 0.51 | 0.74 | 0.60 | 0.82 | 0.18 | 0.26 | 0.25 | 0.35 | 0.19 | 0.26 | 0.036 | 0.032 |
| $\begin{aligned} & S=8 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=250$ | 0.32 | 0.40 | 0.36 | 0.31 | 0.38 | 0.67 | 0.62 | 0.70 | 0.98 | 0.73 | 0.83 | 0.83 | 0.64 | 0.63 | 0.66 | 0.54 | 0.85 | 0.49 | 0.67 | 0.082 | 0.064 |
|  | $L=40, N=1000$ | 0.23 | 0.25 | 0.23 | 0.24 | 0.25 | 0.61 | 0.89 | 0.58 | 1.26 | 0.67 | 0.74 | 0.73 | 0.74 | 0.33 | 0.43 | 0.36 | 0.50 | 0.30 | 0.38 | 0.060 | 0.055 |
|  | $L=160, N=250$ | 0.17 | 0.20 | 0.17 | 0.18 | 0.18 | 0.62 | 0.60 | 0.68 | 1.01 | 0.55 | 0.79 | 0.64 | 0.69 | 0.42 | 0.44 | 0.37 | 0.58 | 0.26 | 0.33 | 0.042 | 0.044 |
|  | $L=160, N=1000$ | 0.10 | 0.13 | 0.12 | 0.13 | 0.14 | 0.35 | 0.54 | 0.46 | 0.83 | 0.43 | 0.58 | 0.45 | 0.60 | 0.19 | 0.24 | 0.21 | 0.29 | 0.14 | 0.21 | 0.035 | 0.030 |
| $\begin{aligned} & S=2 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 1.61 | 2.13 | 2.23 | 2.02 | 2.34 | 2.50 | 3.22 | 2.47 | 6.00 | 3.06 | 2.65 | 3.14 | 2.48 | 1.71 | 2.03 | 1.68 | 2.22 | 1.62 | 1.99 | 0.068 | 0.063 |
|  | $L=40, N=1000$ | 1.79 | 1.70 | 1.68 | 1.81 | 1.75 | 1.97 | 2.88 | 2.43 | 3.49 | 2.80 | 3.78 | 2.49 | 3.88 | 0.87 | 1.08 | 0.96 | 1.35 | 1.24 | 1.56 | 0.065 | 0.061 |
|  | $L=160, N=250$ | 0.78 | 1.07 | 0.91 | 0.97 | 1.11 | 1.50 | 2.39 | 2.02 | 3.98 | 2.01 | 3.07 | 2.15 | 2.78 | 0.83 | 1.19 | 0.96 | 1.24 | 0.82 | 1.03 | 0.036 | 0.032 |
|  | $L=160, N=1000$ | 0.94 | 0.94 | 0.91 | 1.01 | 0.82 | 1.03 | 1.55 | 1.32 | 3.11 | 1.46 | 3.28 | 1.81 | 3.31 | 0.40 | 0.62 | 0.53 | 0.94 | 0.55 | 0.61 | 0.033 | 0.031 |
| $\begin{aligned} & S=8 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 0.81 | 0.89 | 0.89 | 0.99 | 0.86 | 2.79 | 3.10 | 3.71 | 3.95 | 2.34 | 3.33 | 3.15 | 3.37 | 1.54 | 1.98 | 1.74 | 2.00 | 1.20 | 1.59 | 0.065 | 0.049 |
|  | $L=40, N=1000$ | 0.67 | 0.83 | 0.67 | 0.65 | 0.81 | 1.82 | 2.41 | 2.24 | 3.05 | 2.04 | 2.43 | 2.30 | 3.06 | 0.92 | 1.13 | 1.00 | 1.31 | 0.89 | 1.06 | 0.061 | 0.060 |
|  | $L=160, N=250$ | 0.47 | 0.44 | 0.44 | 0.52 | 0.53 | 1.58 | 2.41 | 2.10 | 3.27 | 1.43 | 2.71 | 1.76 | 2.52 | 0.86 | 1.18 | 0.93 | 1.34 | 0.61 | 0.66 | 0.033 | 0.027 |
|  | $L=160, N=1000$ | 0.32 | 0.38 | 0.41 | 0.34 | 0.38 | 1.08 | 1.37 | 1.50 | 2.52 | 1.13 | 1.87 | 1.41 | 1.87 | 0.49 | 0.57 | 0.59 | 0.83 | 0.37 | 0.49 | 0.034 | 0.028 |
| Tree U |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\theta=0.0025$ | $L=40, N=1000$ | 0.48 | 0.53 | 0.44 | 0.52 | 0.46 | 0.68 | 0.74 | 0.64 | 0.83 | 0.81 | 0.71 | 0.73 | 0.77 | 0.57 | 0.40 | 0.38 | 0.33 | 0.39 | 0.53 | 0.081 | 0.059 |
|  | $L=160, N=250$ | 0.30 | 0.36 | 0.37 | 0.40 | 0.38 | 0.65 | 0.71 | 0.57 | 0.97 | 0.85 | 0.79 | 0.74 | 1.30 | 0.51 | 0.39 | 0.36 | 0.33 | 0.37 | 0.40 | 0.063 | 0.039 |
|  | $L=160, N=1000$ | 0.19 | 0.26 | 0.28 | 0.28 | 0.22 | 0.50 | 0.57 | 0.43 | 0.67 | 0.59 | 0.94 | 0.49 | 0.78 | 0.30 | 0.26 | 0.20 | 0.21 | 0.20 | 0.24 | 0.043 | 0.032 |
| $\begin{aligned} & S=8 \\ & \theta=0.0025 \end{aligned}$ | $L=40, N=250$ | 0.35 | 0.38 | 0.36 | 0.38 | 0.37 | 0.55 | 0.67 | 0.69 | 1.00 | 0.70 | 0.96 | 0.65 | 0.75 | 0.74 | 0.66 | 0.63 | 0.54 | 0.60 | 0.60 | 0.116 | 0.068 |
|  | $L=40, N=1000$ | 0.23 | 0.24 | 0.26 | 0.23 | 0.24 | 0.74 | 0.77 | 0.62 | 1.03 | 0.80 | 0.86 | 0.69 | 0.90 | 0.42 | 0.39 | 0.35 | 0.36 | 0.26 | 0.37 | 0.067 | 0.062 |
|  | $L=160, N=250$ | 0.16 | 0.18 | 0.19 | 0.18 | 0.17 | 0.69 | 0.74 | 0.58 | 0.85 | 0.70 | 1.03 | 0.54 | 0.74 | 0.44 | 0.36 | 0.40 | 0.31 | 0.30 | 0.28 | 0.057 | 0.032 |
|  | $L=160, N=1000$ | 0.11 | 0.14 | 0.12 | 0.12 | 0.12 | 0.53 | 0.49 | 0.37 | 0.62 | 0.42 | 0.61 | 0.33 | 0.59 | 0.32 | 0.22 | 0.19 | 0.21 | 0.13 | 0.20 | 0.033 | 0.030 |
| $\begin{aligned} & S=2 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 1.77 | 2.17 | 2.11 | 2.06 | 1.93 | 2.58 | 2.92 | 2.62 | 4.39 | 3.05 | 3.00 | 2.67 | 2.98 | 2.09 | 1.66 | 1.48 | 1.46 | 1.60 | 2.29 | 0.090 | 0.056 |
|  | $L=40, N=1000$ | 1.49 | 2.09 | 1.83 | 1.97 | 1.75 | 2.52 | 2.40 | 2.12 | 3.02 | 3.23 | 2.81 | 2.87 | 3.61 | 1.24 | 1.17 | 0.88 | 0.93 | 1.17 | 1.38 | 0.068 | 0.059 |
|  | $L=160, N=250$ | 0.85 | 1.09 | 1.11 | 1.08 | 0.94 | 2.19 | 2.17 | 1.80 | 3.23 | 2.50 | 2.84 | 2.07 | 3.05 | 1.23 | 1.01 | 0.97 | 1.03 | 0.90 | 1.02 | 0.045 | 0.027 |
|  | $L=160, N=1000$ | 0.75 | 0.89 | 0.84 | 0.98 | 0.92 | 1.32 | 1.47 | 1.27 | 1.88 | 1.77 | 2.67 | 1.46 | 2.74 | 0.67 | 0.59 | 0.44 | 0.53 | 0.53 | 0.57 | 0.034 | 0.026 |
| $\begin{aligned} & S=8 \\ & \theta=0.01 \end{aligned}$ | $L=40, N=250$ | 0.85 | 0.98 | 1.08 | 0.96 | 0.96 | 2.83 | 3.04 | 2.61 | 3.64 | 2.91 | 3.54 | 2.56 | 2.93 | 1.96 | 1.31 | 1.41 | 1.39 | 1.14 | 1.39 | 0.074 | 0.058 |
|  | $L=40, N=1000$ | 0.75 | 0.67 | 0.70 | 0.78 | 0.69 | 2.39 | 2.60 | 2.27 | 3.24 | 2.32 | 2.97 | 2.35 | 2.86 | 1.18 | 0.99 | 0.94 | 1.00 | 0.84 | 0.97 | 0.057 | 0.052 |
|  | $L=160, N=250$ | 0.44 | 0.52 | 0.49 | 0.46 | 0.47 | 2.46 | 2.10 | 1.56 | 2.64 | 1.75 | 3.16 | 1.45 | 2.27 | 1.38 | 1.04 | 0.74 | 0.91 | 0.64 | 0.76 | 0.040 | 0.034 |
|  | $L=160, N=1000$ | 0.35 | 0.38 | 0.33 | 0.39 | 0.36 | 1.30 | 1.28 | 1.02 | 2.20 | 1.31 | 2.62 | 1.14 | 2.09 | 0.63 | 0.52 | 0.42 | 0.54 | 0.41 | 0.43 | 0.031 | 0.028 |

